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Earth's Gravity Field Mapping Requirements and Concept

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EARTH'S GRAVITY FIELD MAPPING REQUIREMENTS AND CONCEPT

F. O. Vonbun and W. D. Kahn

ABSTRACT

The objective of this paper is to discuss a future sensor (gradiometer) for mapping the earth's gravity field to meet future scientific and practical requirements for earth and oceanic dynamics. These are approximately ± 0.1 to 10 mgal over a block size of about 50 km and over land and an ocean geoid to 1 to 2 cm over a distance of about 50 km.

To achieve these values requires a gravity gradiometer with a sensitivity of approximately 10^{-4} EU in a circular polar orbiting spacecraft with an orbital altitude ranging 160 km to 180 km.

EARTH'S GRAVITY FIELD MAPPING REQUIREMENTS AND CONCEPT

INTRODUCTION: GRAVITY ANOMALY REQUIREMENTS

The observation of spacecraft orbits started a revolution in the determination of the "shape" of our planet. (Ref. 1). As time has progressed, so have the demands for gravity information. Spacecraft orbital perturbation analyses were and still are the principal means for deriving the earth gravity field. The best gravity field model published in the open literature to date may be the GEM 10B (Ref. 2), a field with a block size resolution of about 550 km ($5^\circ \times 5^\circ$) and an error of about 10 mgal on the average.

It is well known that the earth's gravity field is a result of the distribution of its mass and thus is the fundamental base needed to construct an accurate geoid. The latter in turn is the steady state surface of the ocean if there were no tides, currents, wind pile-ups and atmospheric pressure changes. On the other hand, it is these changes which can tell us about the ocean currents if a reference surface (geoid) is known to an accuracy of 1 to 2 cm and a spatial resolution of approximately 50 km and larger (Ref. 3). It is this small resolution which dictates a new approach to the determination of the gravity field.

At present the best approach to increasing our knowledge of the earth's gravity field is the use of Satellite-to-Satellite Tracking (SST), which was started at Goddard Space Flight Center (GSFC) during the late 1960's. These techniques demonstrated experimentally with ATS-6 and Nimbus E, with ATS-6 and GEOS-3 (Ref. 4), and finally with ATS-6 and Apollo-Soyuz (Ref. 5) which can be considered the first demonstration of a low-flying type gravity satellite mission. This effort has now culminated in GRAVSAT-A (Ref. 3 and 6), a possible 1983 new program start candidate as an Applications Spacecraft essential for future studies of the dynamics of the solid earth and the oceans.

The scientific requirements for future studies of earth and ocean dynamics can be summarized as follows (Ref. 3):

- Solid Earth Physics: ± 0.1 to 1.0 mgal; 50 to 60 km resolution
- Ocean Circulation: ± 1 to 2 cm; 50 to 60 km resolution

These requirements can be compared with our present knowledge:

- Solid Earth Physics: 10 mgal; 500 km resolution.
- Ocean Circulation: 200 cm geoid; 500 km resolution.
- Local Ocean areas: 20 mgal; 100 km resolution.
50 cm; 100 km resolution.

MISSION OBJECTIVES

The objectives of the GRAVSAT-A mission to satisfy the above requirements can now be stated as follows:

- Provide an accurate gravity field to study the structure of the crust and upper mantle of the earth.
- Provide an accurate global geoid to be used as the reference surface for studies of ocean currents.

CONCEPT

Sensor Systems

As has been mentioned, the use of SST will permit resolution of gravity field anomalies with an accuracy of about ± 1 mgal for each $1^\circ \times 1^\circ$ area of the globe, the present goal and capability for GRAVSAT-A. However, this seems to be the limit of the SST technique as shown below.

If anomaly resolution is to be universal for a block size of approximately 50×50 km another "sensor" must be developed and used. A gravity gradiometer with a sensitivity of 10^{-4} EU can achieve these more stringent requirements.

In the following, the relationship between the two possible "sensors," for mapping the gravity field, i.e., SST and a gradiometer (Refs. 7, 8) is shown. The reason for considering the gradiometer is that it can resolve the short wavelength components of the earth's gravity field up to degree and order 400.

In order to estimate and compare the expected "Signals," we shall analyze their respective power spectra (Refs. 7, 8, 9). The earth's gravity potential is given by (e.g. Ref. 9)

$$V = \frac{GM}{r} \left[1 + \sum_{\ell=2}^N \sum_{m=0}^{\ell} \left(\frac{a}{r} \right)^{\ell} P_{\ell m}(\sin\phi) [\bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda] \right], \quad (1)$$

where

r is the geocentric satellite distance,

ϕ is the sub-satellite geocentric latitude,

λ is the sub-satellite east longitude,

a is the mean radius of the earth,

$\bar{P}_{\ell m}(\sin\phi)$ are the fully normalized associated Legendre polynomials of degree ℓ and order m , and

$\bar{C}_{\ell m}, \bar{S}_{\ell m}$ are the fully normalized gravitational coefficients.

These power spectra will now be derived for the SST and the gradiometer systems.

Satellite-to-Satellite Tracking System (SST)

The SST system measures the range rate between two spacecraft (Ref. 6) resulting from the difference of gravity acting upon the two spacecraft. The total energy of a spacecraft in orbit is given by:

$$E = \frac{1}{2} mv^2 + T \quad (2)$$

where v is its speed of the spacecraft, m its mass and T is the disturbing potential of the earth. The variation of T is then

$$\delta T = -v\delta v. \quad (3)$$

From equation (1) without the central force term, together with equation (3) we obtain

$$\delta v = v \sum_{\ell=2}^N \sum_{m=0}^{\ell} \left(\frac{a}{r} \right)^{\ell} \bar{P}_{\ell m}(\sin \phi) (\bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda). \quad (4)$$

The velocity power spectrum of δv is now obtained by squaring equation (4) and integrating over the unit sphere:

$$V_{\ell}^2(\delta v) = v^2 \left(\frac{a}{r} \right)^{2\ell} \sum_{m=0}^{\ell} (\bar{C}_{\ell m}^2 + \bar{S}_{\ell m}^2) (\text{cm/s})^2, \quad (5)$$

where Kaula's rule (Ref. 10), the summation term can be replaced with

$$(2\ell + 1) \left(\frac{10^{-5}}{\ell^2} \right)^2.$$

The square root of the power spectrum, that is the expected velocity signal for each wave number reads:

$$S_{\ell}(\delta v) = \left(\frac{GM}{a+h} \right)^{1/2} \left(\frac{a}{a+h} \right)^{\ell} (2\ell + 1)^{1/2} \left(\frac{10^{-5}}{\ell^2} \right) \quad (6)$$

where

$h \equiv$ spacecraft orbital height,

$a \equiv$ mean radius of the earth (i.e. nearly circular orbits are assumed for the missions discussed), and

$GM \equiv$ gravitational parameter of the earth.

introducing $GM \doteq 4 \times 10^{20} \text{ cm}^3/\text{s}^2$, assuming $\ell \geq 40$, and converting to $\mu\text{m/s}$ results in:

$$S_{\ell}(\delta v) = 2.82 \times 10^9 (a+h)^{-1/2} \left(\frac{a}{a+h} \right)^{\ell} (\ell)^{-3/2} (\mu\text{m/s}) \quad (7)$$

for a and h expressed in cm.

These spectral values are described in more detail and compared with experimental data in the next section.

COMPARISON WITH THE APOLLO SOYUZ EXPERIMENT

Satellite-to-Satellite Range Rate Signals

As was stated earlier, an actual satellite-to-satellite tracking (SST) experiment (Ref. 5) has demonstrated that the recovery of gravity anomalies is feasible. In the later Apollo experiment the measurement noise of the ATS-6 to Apollo link (high-low concept) was determined to be about 300 $\mu\text{m/s}$. The next step is to relate this experimental value of 300 $\mu\text{m/s}$ to the value predicted by equation (6). That is, for the Apollo orbital altitude of 230 km and a 550-km horizontal resolution ($\ell = 36$), the signal amounts to

$$S_{36}(\delta v) = 11 \left(\frac{5378}{6608} \right)^{36} \left(\frac{10^4}{36^{3/2}} \right) = 140 \mu\text{m/s}. \quad (8)$$

This, however, represents a global value. Since the experiment was conducted over a local region, a gravity signal increase was to be expected.

Gravity Anomaly Uncertainties

One of the important quantities to be estimated for SST techniques is the uncertainty attributable to the technique as was discussed before. A theoretical estimate of the errors in terms of the recovered gravity anomalies can only be obtained by simulation studies which are in general tedious and costly. However, if experimental values are available, simple scaling of these values can be applied for error estimation. In this case, the Apollo-Soyuz errors will be used. In order to obtain the scaling law, we must express the gravity anomalies in terms of the disturbing potential T (Ref. 9), that is

$$\delta g = - \frac{\partial T}{\partial r} - \frac{2}{r} T. \quad (9)$$

In spherical harmonics, equation (9) reads as follows:

$$\delta g = \frac{GM}{r^2} \sum_{\ell=2}^{\ell=N} \sum_{m=0}^{m=\ell} (\ell-1) \left(\frac{a}{r}\right)^{\ell} \bar{P}_{\ell m}(\sin(\phi)) \cdot (\bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda). \quad (10)$$

An estimation of this increase (noise decrease) is obtained by adjusting the global value, i.e., by multiplying the signal by the ratio of the global surface area to the local experiment area. This problem is treated in detail by E. H. Gaposchkin (Ref. 12). For the Apollo experiment this value is about 3 (Ref. 5).

Applying this process to equation (8) yields a signal of about $320 \mu\text{m/s}$. Assuming further that there were an average two cross-overs per $5 \times 5^\circ$ block and that the signal is noise limited, the signal is a factor of $\sqrt{2}$ larger, that is $450 \mu\text{m}$. It is assumed that the signal used in the computation is a composite, in the least-square sense, of all the signals detected over the experimental area. Since the predicted single spectral signal is about $140 \mu\text{m/s}$ and the "useful" signal was about $450 \mu\text{m}$, the computed signal-to-noise ratio was found to be about 3, making a gravity estimation possible. The analogous expression to equations (4) and (5) for the power spectral value of δg is

$$V_{\ell}^2(\delta g) = \left(\frac{GM}{r^2}\right)^2 (\ell-1)^2 \cdot \left(\frac{a}{r}\right)^{2\ell} \sum_{m=0}^{\ell} (\bar{C}_{\ell m}^2 + \bar{S}_{\ell m}^2). \quad (11)$$

Simplifying equation (11) by assuming $\ell \gg 1$ and replacing the sum of the normalized coefficients of the harmonic expression by Kaula's rule (Ref. 10) one obtains the gravity anomaly signal

$$S_{\ell}(\delta g) = \sqrt{2} \frac{GM}{(a+h)^2 \ell^{1/2}} \left(\frac{a}{a+h}\right)^{\ell} \times 10^{-5} \text{ cm/s}^2. \quad (12)$$

Using equation (12), one can now form the needed ratios representing the desired scaling law:

$$\frac{S_{\ell_2}(\delta g_2)}{S_{\ell_1}(\delta g_1)} = \left[\frac{a+h_1}{a+h_2}\right]^2 \left(\frac{\ell_1}{\ell_2}\right)^{1/2} \left(\frac{a}{a+h_1}\right)^{-\ell_1} \left(\frac{a}{a+h_2}\right)^{\ell_2} \left(\frac{N_1}{N_2}\right)^{1/2}, \quad (13)$$

where subscripts 1 and 2 refer to Apollo and Gravsat respectively,

$$h_1 = 230 \times 10^5 \text{ cm},$$

$$h_2 = 160 \times 10^5 \text{ cm},$$

$$\ell_1 = 36, \text{ corresponding to a } (550 \times 550) \text{ km}^2 \text{ block size},$$

$$\ell_2 = 200, \text{ corresponding to a } (100 \times 100) \text{ km}^2 \text{ block size},$$

$$V_{\ell_1}(\delta g_1) \equiv 10 \text{ mgal (R.M.S. associated with recovered } (550)^2 \text{ km}^2 \text{ anomalies from Apollo based on comparison with GEM 10B,}$$

$$V_{\ell_2}(\delta g_2) = \text{predicted rms for Gravsat } (100 \times 100) \text{ km}^2 \text{ anomaly,}$$

$$N_1 = \text{number of orbit revolutions available for Apollo derived anomalies } (N_1 = 2),$$

$$N_2 = \text{number of orbit revolutions expected for Gravsat derived anomalies } (N_2 = 46),$$

$$a = \text{mean earth radius } (6.4 \times 10^8 \text{ cm}) \text{ and}$$

$$GM = 4 \times 10^{20} \text{ cm}^3/\text{sec}^2.$$

We now form the ratio of equation (6) to Equation (12):

$$\frac{V_{\ell}(\delta v)}{V_{\ell}(\delta g)} = \frac{10(a+h)^{3/2}}{(GM)^{1/2}\ell} \mu\text{m/s/mgal} \quad (14)$$

Combining (13) and (14) then yields

$$S_{\ell_2}(\delta v_2) = \frac{10(a+h_2)^{3/2}}{(GM)^{1/2}\ell_2} \left(\frac{a+h_1}{a+h_2}\right)^2 \left(\frac{\ell_1}{\ell_2}\right)^{1/2} \left(\frac{a}{a+h_1}\right)^{-\ell_1} \left(\frac{a}{a+h_2}\right)^{\ell_2} \left(\frac{N_1}{N_2}\right)^{1/2} S_{\ell_1}(\delta g_1). \quad (15)$$

Using the values specified above in equation (15), one obtains an estimate of the precision required for the Gravsat-A sensor, that is

$$S_{\ell_2}(\delta v_2) = 0.8 \mu\text{m/s}.$$

This value agrees quite well with the precision specified for Gravsat-A, $1 \mu\text{m/s}$ (Ref. 3).

From equation (15) it can be seen that only a moderate improvement or reduction of error in δg can be obtained by lowering the Gravsat altitude h_2 (N_2 is increased correspondingly). For example, lowering the orbital altitude from 160 km to 150 km changes the required sensitivity by only a factor of 1.4; but such a step introduces serious engineering problems (more fuel, tighter control, thermal problems, etc.).

GRADIOMETER SYSTEM

Using an approach similar to that used for satellite-to-satellite tracking (SST) we can obtain spectrum for the gravity gradient sensed by a super cooled gradiometer system as follows: the second derivative of the disturbing potential T with respect to the radius, $\partial^2 T / \partial r^2$, is the radial component of the gravity field tensor. This component is the largest in magnitude of the nine components of the gravity tensor. Forming the ratio of the radial to the along-track and cross-track components of the field tensor one obtains (Ref. 7).

$$\frac{V_{\ell \text{ rad}}^2 (\text{grad})}{V_{\ell \text{ cross}}^2 (\text{grad})} = \frac{V_{\ell \text{ rad}}^2 (\text{grad})}{V_{\ell \text{ along}}^2 (\text{grad})} \approx 4. \quad (17a)$$

The other ratios, involving the cross-components are very much smaller. Thus the spectrum of the main component is used here for estimating the expected signal for the gravity gradiometer.

The radial component of the gravity field tensor is

$$\frac{\partial^2 T}{\partial r^2} = \frac{GM}{r^3} \sum_{\ell=2}^N \sum_{m=0}^{\ell} (\ell+1)(\ell+2) \left(\frac{a}{r}\right)^{\ell} \bar{P}_{\ell m}(\sin \varphi) \left\{ \bar{C}_{\ell m} \cos m\lambda + \bar{S}_{\ell m} \sin m\lambda \right\} \quad (17b)$$

The power spectrum by degree ℓ is given by the square of the sum over all the m 's, that is

$$V_{\ell}^2 (\text{grad}) = \left(\frac{GM}{r^3}\right)^2 (\ell+1)^2 (\ell+2)^2 \left(\frac{a}{r}\right)^{2\ell} \sum_{m=0}^{\ell} (\bar{C}_{\ell m}^2 + \bar{S}_{\ell m}^2). \quad (18)$$

For $\ell \geq 40$, the signal in simplified form is:

$$S_{\ell} (\text{grad}) = (2 \times 10^{-10})^{1/2} \frac{GM}{(a+h)^3} \left(\frac{a}{a+h}\right)^{\ell} (\ell)^{1/2} \frac{\text{cm/s}^2}{\text{cm}}. \quad (18a)$$

Introducing the value for GM , we obtain the "signal" by degree in terms of EU:

$$S_{\ell} (\text{grad}) = \frac{5.64 \times 10^{24}}{(a+h)^3} \left(\frac{a}{a+h}\right)^{\ell} (\ell)^{1/2} (\text{EU}), \quad (19)$$

(1EU = 10^{-9} cm/s²/cm).

For the proper numerical values of the expected signals one obtains from (7) and (18a) respectively

$$S_{\ell}(\delta v) \doteq (11 \times 10^4) \left(\frac{a}{a+h} \right)^{\ell} (\ell)^{-3/2} \mu\text{m/s} \quad (20a)$$

and

$$S(\text{grad}) \doteq (2 \times 10^8)^{1/2} \left(\frac{g}{a+h} \right) \left(\frac{a}{a+h} \right)^{\ell+2} (\ell)^{1/2} \text{ EU} \quad (20b)$$

where

a = the mean radius of the earth, 4×10^3 km

h = the orbital height of the spacecraft, 160 km and 180 km,

g = acceleration of gravity, 981 cm/s^2 , and

ℓ = the degree of the spherical harmonic expansion of the earth gravity field.

As can be seen, the gravity field attenuation factor $(a/a+h)^{\ell}$ is the same in each case. The quantity $(a/a+h)^{\ell}$ for $h < a$, as is always the case for gravity type missions, is near unity (0.95). What is interesting, however, is that the resolution ℓ (degree of the harmonic expansion) has a drastically different effect on the expected "gravity signals" in favor of the gradiometer for $\ell > 150$. In terms of block size b : where

$$b = \frac{2\pi a}{2\ell} \doteq \frac{20,000}{\ell} \text{ km}.$$

Figures 1 and 2 depict the expected gravity spectral signals as sensed by the SST and gravity gradient sensors for orbital altitudes of 160 and 180 km respectively. A relationship can be obtained from forming the square root of the ratio of equations (5) and (18) that is

$$S_{\ell_1}(\delta v) = (a+h_2)^3 [GM(a+h_1)]^{-1/2} \left(\frac{a}{a+h_1} \right)^{\ell_1} \left(\frac{a}{a+h_2} \right)^{-\ell_2} \left(\frac{\ell_1}{\ell_2} \right)^{1/2} \left(\frac{1}{\ell_1^2} \right) S_{\ell_2}(\text{grad}) \quad (21)$$

Assuming $h_1 = h_2$ and $\ell_1 = \ell_2$ equation (21) reads

$$S_{\ell}(\delta v) \text{ cm/s} = \left[\frac{(a+h)^5}{GM} \right]^{1/2} \left(\frac{1}{\ell^2} \right) S_{\ell}(\text{grad}) (\text{cm/s}^2/\text{cm})$$

Introducing $\mu\text{m/s}$ and EU, one obtains

$$S_{\ell}(\delta v) \mu\text{m/s} \approx S_{\ell}(\text{grad}) \left\{ (a+h)^{3/2} \left(\frac{10^{-5}}{\ell^2} \right) \right\} \text{EU} \quad (21a)$$

For $h = 160 \text{ km}$ and $\ell = 200$,

$$S_{\ell}(\delta v) \mu\text{m/s} = 137 S_{\ell}(\text{grad}) \text{EU}. \quad (21b)$$

One $\mu\text{m/s}$ thus corresponds to about 7.10^{-3} EU .

In order to make a meaningful assessment of the above spectral signals the wave number spectrum is integrated for different intervals of degree ℓ .

This provides a measure of the modelled signal above a certain degree. Integrating Equations 20a and 20b yields

$$S_{\ell_1 \ell_2}(\delta v) = \int_{\ell_1}^{\ell_2} S_{\ell}(\delta v) d\ell \quad (22a)$$

For the SST concept and

$$S_{\ell_1 \ell_2}(\text{grad}) = \int_{\ell_1}^{\ell_2} S_{\ell}(\text{grad}) d\ell \quad (22b)$$

for the gradiometer apparent.

Tables 1 & 2 show the signal strength for selected intervals of degree ℓ .

Table 1
SST Wave Number Spectrum

$h \text{ (km)}$	ℓ_1	ℓ_2	$S_{\ell_1 \ell_2}(\delta v) \mu\text{m/sec}$
160	36	100	3.6×10^3
	100	180	2.3×10^2
	180	360	16.4×10^0
	360	1000	7.5×10^{-2}
180	36	100	3.1×10^3
	100	280	1.6×10^2
	180	360	8.6×10^0
	360	1000	2.3×10^{-2}

Table 2
Gradiometer Wave Number Spectrum

h (km)	ℓ_1	ℓ_2	$S_{\ell_1 \ell_2}$ (grad) EU
160	36	100	2.05
	100	180	0.67
	180	360	0.14
	360	1000	2×10^{-3}
180	36	100	1.68
	100	180	0.45
	180	360	0.07
	360	1000	6×10^{-4}

The values tabulated show that comparatively large signals exist compared to the systems mentioned above (i.e., $1 \mu\text{m/s}$, 10^{-4} EU).

For instance for $180 \leq \ell \leq 360$ the signal is about $16 \mu\text{m/s}$ and 1400×10^{-4} EU for orbital height of 160 km.

This demonstrates that the high frequency components of the gravity field exhibit a signal to noise ratio of 14 to 1 and 1400 to 1 respectively. This underscores the power of a gradiometer type space mission.

As can be seen from Figures 1 and 2, and Tables 1 and 2, a gradiometer with a sensitivity of 10^{-4} EU can achieve a resolution of around 50 to 60 km.

Such a system is in the development stage under a NASA grant at the University of Maryland (Figure 3). Figures 4 and 5 are extensions of Figures 1 and 2, respectively. These show clearly the resolution limits of both spaceborne systems even if the systems' sensitivities are improved by several orders of magnitude; the last number representing the ultimate limit for a supercooled (4°K) gradiometer sensor according to Dr. Paik (Ref. 11).

As can be seen a sensitivity change of the gradiometer of four orders of magnitude (10^{-4} to 10^{-7} EU) results only in a resolution change of a factor of two (from 60 km to 30 km). This in

essence means that a gradiometer with a sensitivity of, say, 10^{-4} EU constitutes a practical limit.

Any further improvement in the sensitivity will produce negligible results as far as spatial resolution is concerned.

We next discuss the effect of the gradiometer precision on the gravity anomaly error. We first form the ratio of equations (18) and (11); that is

$$\frac{S_g(\delta g)}{S_g(\text{grad})} = \left(\frac{a+h}{\ell} \right) \text{cm}. \quad (22)$$

Then, scaling to EU and mgal yields

$$S_g(\delta g) (\text{mgal}) = 10^{-6} \left(\frac{a+h}{\ell} \right) S_g(\text{grad}) \text{EU}, \quad (23)$$

where ℓ is nondimensional and a and h are in cm.

Equation (23) states clearly that an increase in the gradiometer precision does substantially decrease the error in the estimated gravity field. For example, using gradiometer orbiting at 160 km, and a $1^\circ \times 1^\circ$ gravity anomaly ($\ell = 200$), one obtains from (23)

$$S_g(\delta g) \text{mgal} = 3 S_g(\text{grad}) \text{EU} \quad (24)$$

A gradiometer with a precision of 10^{-4} yields a gravity anomaly error of 3×10^{-4} mgal. This would mean that using such an instrument one can resolve gravity field anomalies of about 50×50 km to less than one μgal . Practically, this value will probably be less than one mgal since computational difficulties (high correlation etc.) will certainly arise. Only a successful but computationally costly simulation will resolve this question.

CONCLUSION

In summary it can be stated that a further improvement in sensitivity and resolution of the earth gravity field can be realized using a supercooled gravity gradiometer instead of satellite-to-satellite tracking system.

The former sensor provides in situ measurements where the latter provides derived field parameters. A spaceborne polar orbiting gradiometer with a precision of 10^{-4} EU operating at an altitude of 160 km can map the gravity field to a precision of say less than one mgal and a horizontal resolution of about 50 km. This further corresponds to a geoid height error to the subcentimeter level, which in turn satisfies almost all future ocean dynamics requirements.

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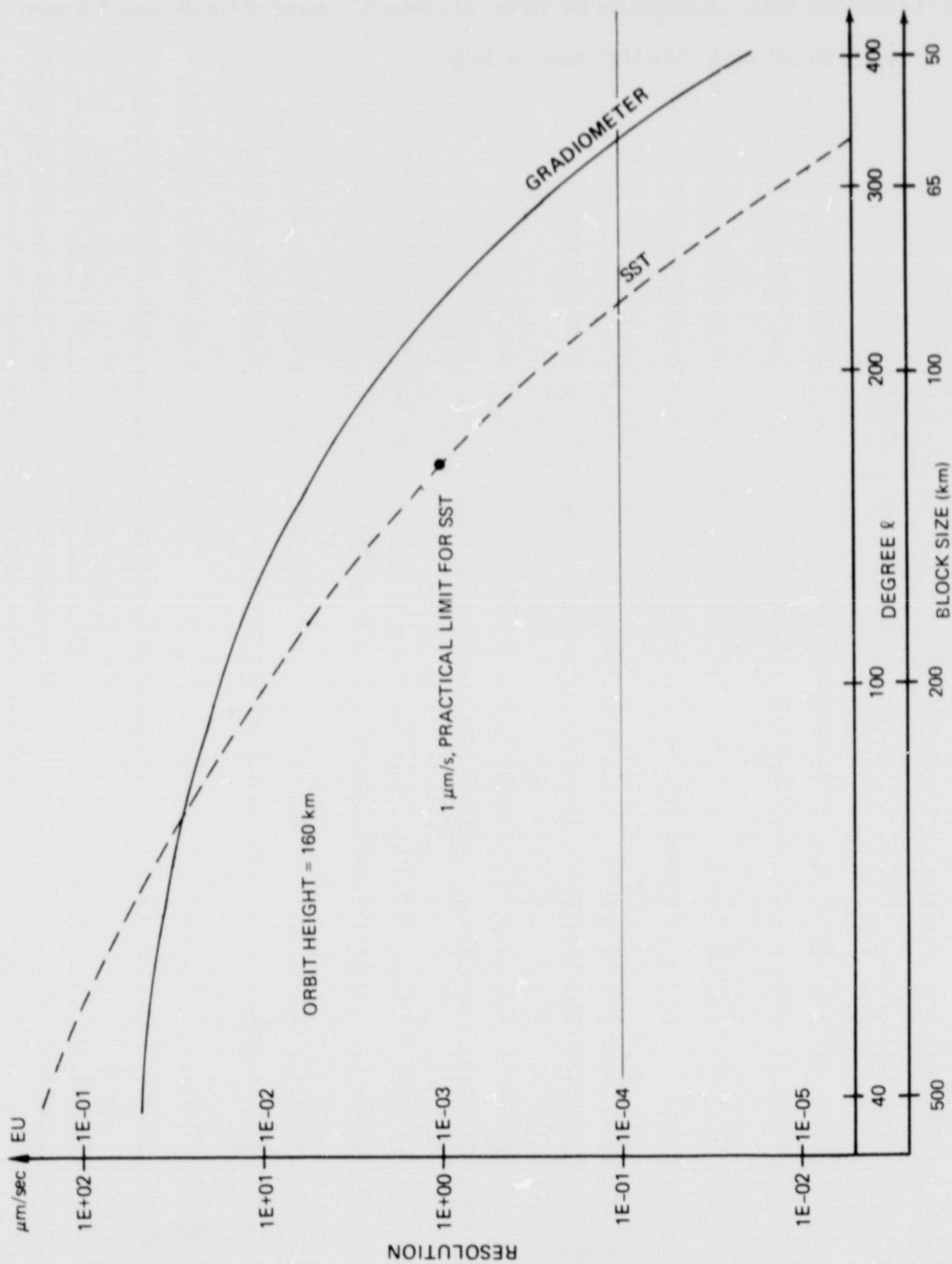


Figure 1. GRAVSAT Spectral Signal Resolution vs Degree for Both Gradiometer and SST Systems.

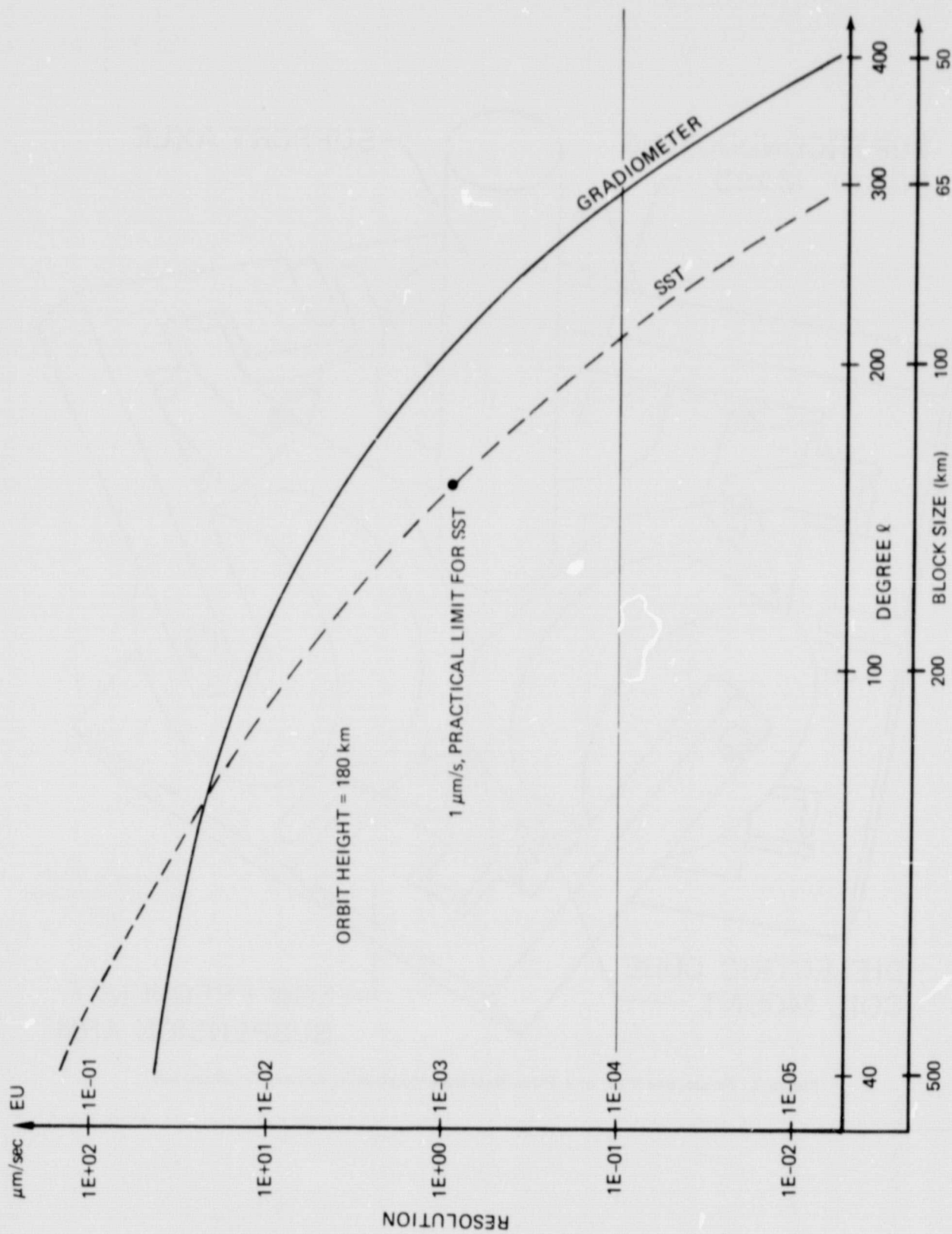


Figure 2. GRAVSAT Spectral Signal Resolution vs Degree for Both Gradiometer and SST Systems.

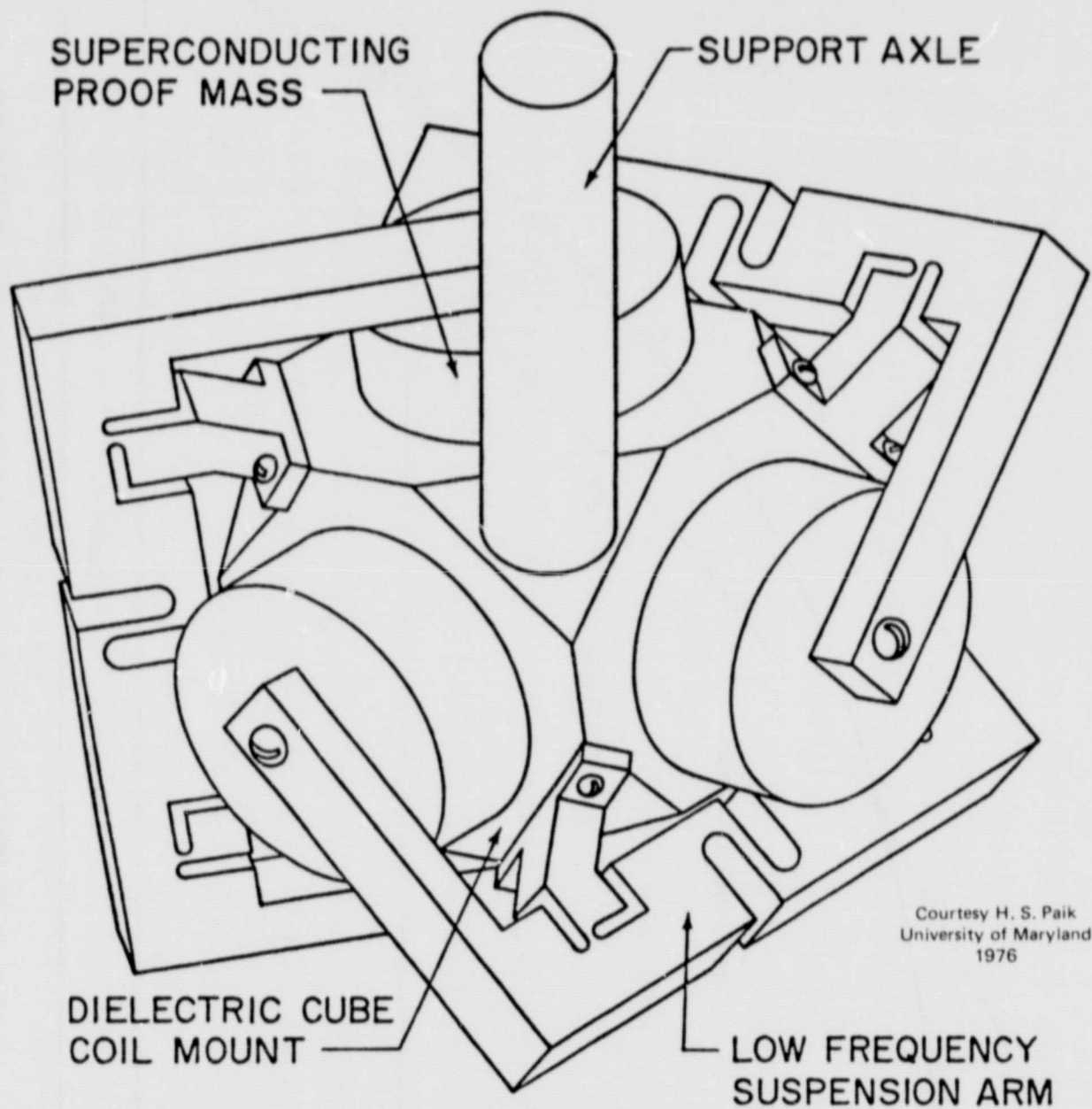


Figure 3. Perspective View of a Three-Axis Superconducting Gravity Gradiometer.

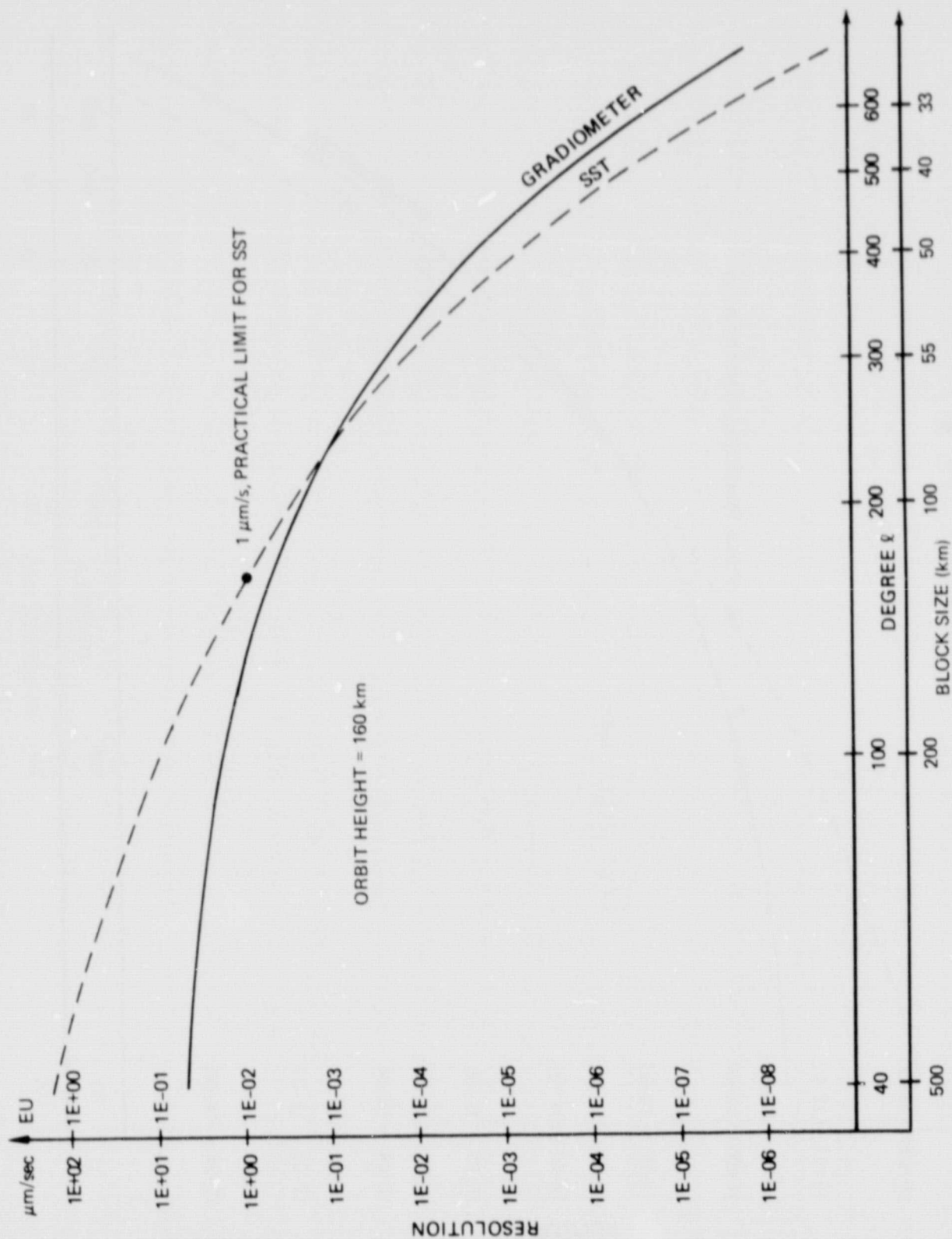


Figure 4. GRAVSAT Spectral System Resolution for Both Gradiometer and SST Systems.

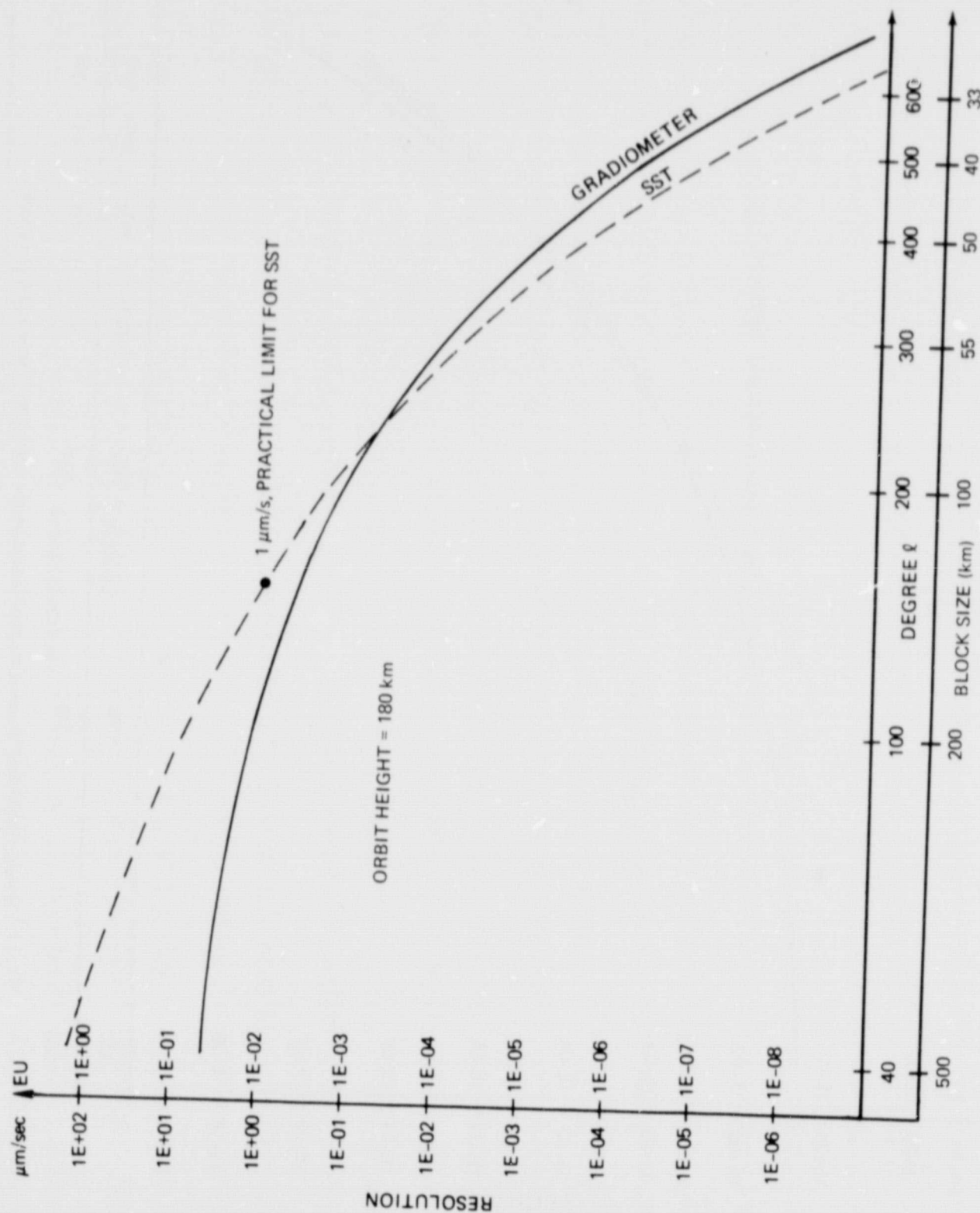


Figure 5. GRAVSAT Spectral Signal Resolution for Both Gradiometer and SST Systems.